1 Units and Constants

$\frac{1\text{J}}{c^2} = 1.11\text{E}{-17}\,\text{kg} = 6.24\text{E}18\,\text{eV}$
$1\text{u} = 1.66\text{E}{-27}\,\text{kg} = 931.49\frac{\text{MeV}}{c^2}$
$1\text{eV} = 1.6\text{E}{-19}\,\text{J} = 1.6\text{E}{-19}\,\text{C V}$
$1\frac{\text{eV}}{c^2} = 1.78\text{E}{-38}\,\text{kg} \qquad 1\frac{\text{eV}}{c} = 5.34\text{E}{-28}\,\text{kg m s}^{-1}$

$h = 6.626\text{E}{-34}\,\text{m}^2\,\text{kg s}^{-1}$ $\quad \hbar = 1.055\text{E}{-34}\,\text{m}^2\,\text{kg s}^{-1}$
$\varepsilon_0 = 8.854\text{E}{-12}\,\text{F m}^{-1}$ $\quad \mu_0 = 1.257\text{E}{-6}\,\text{m kg s}^{-2}\,\text{A}^{-2}$
$k_\text{B} = 1.38\text{E}{-23}\,\text{J K}^{-1}$ $\quad k_\text{e} \equiv (4\pi\varepsilon_0)^{-1} = 8.99\text{E}9\,\text{N m}^2\,\text{C}^{-2}$
$b = 2.897\text{E}{-3}\,\text{K m}$ $\quad \sigma = 5.67\,\text{W K m}^{-1}$
$R_\infty = 1.097\text{E}7\,\text{m}^{-1}$ $\quad a_0 \equiv \frac{\hbar^2}{m_\text{e} k_\text{e} e^2} = 0.0529\,\text{nm}$
$e = 1.602\text{E}{-19}\,\text{C}$ $\quad c_0 = 3.00\text{E}8\,\text{m s}^{-1}$
$m_\text{e} = 4.109\text{E}{-31}\,\text{kg}$ $\quad m_\text{p} = 1.673\text{E}{-27}\,\text{kg}$
$\mu_\text{B} \equiv \frac{e\hbar}{2m_\text{e}} = 9.274\text{E}{-24}\,\text{J T}^{-1}$

2 Special Relativity

$(\Delta s')^2 = (c\Delta t)^2 - (\Delta x)^2 = (\Delta s)^2$

2.1 Galilean Transform

$x' = x - \vec{v}t \quad y' = y \quad z' = z \quad t' = t \quad m' = m \quad \vec{v}'_x = \vec{v}_x - \vec{v}_F$

2.2 Lorentz Transform

$t = \gamma t' \qquad \gamma \equiv \frac{1}{\sqrt{1-\frac{\vec{v}^2}{c^2}}} \qquad \{\frac{\vec{v}}{c} \ll 1\} : \gamma \approx 1 + \frac{\vec{v}^2}{2c^2} \qquad \ell = \frac{\ell'}{\gamma}$
$x' = \gamma(x - \vec{v}t) \qquad y' = y \qquad z' = z \qquad t' = \gamma\left(t - \frac{\vec{v}x}{c^2}\right)$
$\vec{v}'_x = \frac{\vec{v}_x - \vec{v}_F}{1-\frac{\vec{v}_x\vec{v}_F}{c^2}} \qquad \vec{v}'_y = \frac{\vec{v}_y}{\gamma\left(1-\frac{\vec{v}_x\vec{v}_F}{c^2}\right)} \qquad \vec{v}'_z = \frac{\vec{v}_z}{\gamma\left(1-\frac{\vec{v}_x\vec{v}_F}{c^2}\right)}$
$x = \gamma(x' - \vec{v}t') \quad y = y' \quad z = z' \quad t = \gamma\left(t' + \frac{\vec{v}x'}{c^2}\right)$
$\vec{v}_x = \frac{\vec{v}'_x + \vec{v}_F}{1+\frac{\vec{v}'_x\vec{v}_F}{c^2}} \qquad f = \frac{\sqrt{1+\frac{\vec{v}}{c}}}{\sqrt{1-\frac{\vec{v}}{c}}} f'$

2.3 Relativistic Momentum and Energy

$\vec{P} = \gamma m\vec{v} \qquad \vec{F} = \gamma^3 m\vec{a} \qquad E = \gamma mc^2$
$E_\text{k} = E - mc^2 \quad \{\frac{\vec{v}}{c} \ll 1\} : E_\text{k} = \frac{\vec{P}^2}{2m} \quad \vec{v} = c\sqrt{1 - (\frac{E_\text{k}}{mc^2} + 1)^{-2}}$
$E^2 = \vec{P}^2c^2 + (m_0c^2)^2$

3 Electromagnetic Waves

$E_y(x,t) = E_0\cos(kx - \omega t + \phi_0)$
$B_z(x,t) = B_0\cos(kx - \omega t + \phi_0)$
$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f \qquad \vec{v}_\phi = \frac{\omega}{k} = \lambda f = c \qquad T = \frac{1}{f}$
$E = hf = \vec{P}c \qquad \vec{P} = \frac{h}{\lambda} \qquad \lambda = \frac{h}{m\vec{v}}$
$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} \qquad P = \frac{|\vec{S}|^2}{c} = \frac{|E_0|^2}{2\mu_0c^2} \qquad E_\text{e} = \frac{|E_0|^2}{2\mu_0 c}$

3.1 Quantum Theory of Light

$E_\text{e} = \sigma T^4 \varepsilon_\Omega$
$u_f = \frac{8\pi hf^3}{c^3}\left(e^{\frac{hf}{k_\text{B}T}} - 1\right)^{-1}$
Oscillator: $\langle E\rangle = k_\text{B}T \qquad \langle E\rangle = \frac{hf}{e^{\frac{hf}{k_\text{B}T}}-1}$
Photoelectric effect: $E_\text{k,max} = eV_\text{stop} = hf - \phi \qquad f_0 = \frac{\phi}{h}$
Bragg equation: $n\lambda = 2\delta\sin\theta \quad n \in \mathbb{N} \qquad \lambda_\text{min} = \frac{hc}{eV}$
Compton effect: $\lambda' - \lambda_0 = \frac{h}{cm_\text{e}}(1 - \cos\theta)$

$$E'_\gamma = E_\gamma\left[1 + \frac{E_\gamma}{c^2m_\text{e}}(1-\cos\theta)\right]^{-1}$$

4 Matter Particles

$\frac{1}{\lambda} = Z^2R_\infty\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) \qquad E_n = -\frac{k_\text{e}e^2}{2a_0}\frac{Z^2}{n^2} \qquad n \in \mathbb{N}$
$\vec{L} = m_\text{e}\vec{v}R_n = n\hbar \qquad R_n = \frac{a_0n^2}{Z}$

5 Matter Waves

$\lambda = \frac{h}{\vec{P}} \qquad f = \frac{E}{h} \qquad E_\text{k} = \frac{\vec{P}^2}{2m} \qquad \omega = \frac{E}{\hbar} \qquad \vec{P} = \hbar k$
$\vec{v}_\phi = \frac{\omega}{k} = f\lambda = \frac{E}{\vec{P}} = c\sqrt{1 + \left(\frac{mc}{\hbar k}\right)^2}$
$\vec{v}_g = \frac{\text{d}\omega}{\text{d}k}\Big|_{k_0} = \vec{v}_\phi\Big|_{k_0} + k\frac{\text{d}\vec{v}_\phi}{\text{d}k}\Big|_{k_0}$
Fourier integrals: $f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} a(k)e^{ikx}\,dk$
Amp. distribution: $a(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)e^{-ikx}\,dx$
Signal strength: $V(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} g(\omega)e^{i\omega t}\,d\omega$
Spectral content: $g(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} V(t)e^{-i\omega t}\,dt$
$f(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} a(k)e^{i(kx-\omega t)}\,dk$
$\Delta\vec{P}_x\Delta x \geq \frac{\hbar}{2} \qquad \Delta E\Delta t \geq \frac{\hbar}{2} \qquad \Delta k\Delta x \geq \frac{1}{2} \qquad \Delta\omega\Delta t \geq \frac{1}{2}$
$\Psi\Psi^* = |\Psi|^2$
$\Psi_1 \vee \Psi_2 = |\Psi_1|^2 + |\Psi_2|^2$
$\Psi_1 \wedge \Psi_2 = |\Psi_1 + \Psi_2|^2 = |\Psi_1|^2 + |\Psi_2|^2 + 2|\Psi_1||\Psi_2|\cos\phi$

6 Quantum Mechanics in 1 Dimension

$\varphi(x) = |\Psi(x,t)|^2 \qquad |\Psi(x,t)|^2 = \Psi^*(x,t)\Psi(x,t)dx$
$\mathbb{P}(a < x < b) = \int_a^b \varphi(x)dx \qquad \mathbb{P}(-\infty < x < \infty) = 1$
$\Psi_k(x,t) = Ae^{\imath(kx-\omega t)} = A[\cos(kx-\omega t) + \imath\sin(kx-\omega t)]$
$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + E_\text{p}(x)\Psi(x,t) = \imath\hbar\frac{\partial\Psi(x,t)}{\partial t}$
$\{\Psi(x,t) = \psi(x)\phi(t),\ \psi(x) = e^{\imath kx},\ \phi(t) = e^{-\imath\omega t}\}$:
$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + E_\text{p}(x)\psi(x) = E\psi(x)$
$|\Psi(x,t)|^2 = |\psi(x)|^2$

6.1 Particle in a Box

$\psi(x) = A\sin kx + B\cos kx = Ae^{\imath kx} + Be^{\imath kx} \qquad \psi(0), \psi(\ell) = 0$
$E_n = \frac{n^2\pi^2\hbar^2}{2m\ell^2} \qquad \psi_n(0 < x < \ell) = \sqrt{\frac{2}{\ell}}\sin\left(\frac{n\pi x}{\ell}\right) \qquad n \in \mathbb{N}$

6.2 Expectation Values and Operators

Observable: $\langle x\rangle = \int_{-\infty}^{+\infty}\Psi^* x\Psi dx \qquad \sigma = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$
Operator: $\langle X\rangle = \int_{-\infty}^{+\infty}\Psi^*[X]\Psi dx$

Observable	Operator	Observable	Operator
x	x	$\mathscr{H}$	$E_\text{p}(x) - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$
E_k	$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$	$\vec{P}$	$\frac{\hbar}{\imath}\frac{\partial}{\partial x}$
E_p	$E_\text{p}(x)$	E	$i\hbar\frac{\partial}{\partial t}$

7 Tunnelling

$\Psi(x,t)|_{x=0} = \underbrace{Ae^{\imath(kx-\omega t)}}_{\text{incident}} + \underbrace{Be^{\imath(-kx-\omega t)}}_{\text{reflected}}$
$\Psi(x,t)|_{x=\ell} = \underbrace{Ce^{\imath(kx-\omega t)}}_{\text{transmitted}}$
$R \equiv \frac{|B|^2}{|A|^2} \qquad T \equiv \frac{|C|^2}{|A|^2} \qquad R + T = 1$
$T(E) = \frac{1}{1+\frac{E_\text{p}^2}{4E(E_\text{p}-E)}\sinh^2\alpha\ell} \qquad \alpha \equiv \frac{\sqrt{2m(E_\text{p}-E)}}{\hbar} \qquad \delta_\text{pen} = \frac{1}{\alpha}$

7.1 α Decay

$T(E) = e^{\left(-4\pi Z\sqrt{\frac{E_0}{E}} + 8\sqrt{\frac{ZR}{r_0}}\right)} \qquad E_0 \equiv \frac{ke^2}{2r_0} = 0.0993\,\text{MeV}$
$r_0 \equiv \frac{\hbar^2}{m_\alpha ke^2} = 7.25\,\text{fm} \qquad m_\alpha = 7295m_\text{e}$
$\lambda = fT(E) \qquad f = \frac{\vec{v}_\alpha}{2R} = 10\text{E}21\,\text{s}^{-1}$
$N(t) = N_0e^{\frac{-t}{\tau}} = N_0e^{-\lambda t} \qquad \{N/N_0 = 0.5\} : t_{1/2} = \frac{\ln 2}{\lambda}$

8 Quantum Mechanics in 3 Dimensions

8.1 Cartesian Coordinates

Laplacian: $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$
$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + E_\text{p}(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r}) \qquad \mathbf{r} = \langle x_1, x_2, x_3\rangle$

$\{E_\mathrm{p} = 0\} : \psi(\mathbf{r}) = \psi_1(x_1)\psi_2(x_2)\psi_3(x_3) \quad E = \sum_i -\frac{\hbar^2}{2m\psi_i}\frac{\partial^2\psi_i}{\partial x_i^2}$

$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + E_\mathrm{p}(\mathbf{r})\Psi(\mathbf{r},t) = i\hbar\frac{\partial\Psi}{\partial t}$

$\Psi(\mathbf{r},t) = A\sin(k_1x_1)\sin(k_2x_2)\sin(k_3x_3)e^{-\imath\omega t} \qquad 0 < x_i < \ell$

8.1.1 Particle in a Box

$E = \frac{1}{2m}(|\vec{P}_x|^2 + |\vec{P}_y|^2 + |\vec{P}_z|^2) \qquad |\vec{P}_i| = n_i\frac{\pi\hbar}{\ell} \qquad n_i \in \mathbb{N}$

$E = \frac{\pi^2\hbar^2}{2m\ell^2}(n_1^2 + n_2^2 + n_3^2)$

8.2 Spherical Coordinates

$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\left(\frac{\partial^2}{\partial\theta^2} + \cot\theta\frac{\partial}{\partial\theta} + \csc^2\theta\frac{\partial^2}{\partial\phi^2}\right)$

$\mathbf{r} = \langle r,\theta,\phi\rangle \qquad \psi(\mathbf{r}) = R(r)\Theta(\theta)\Phi(\phi)$

Hydrogen-like ion:

$\Psi(r,\theta,\phi,t) = R_{n\ell}(r)Y_\ell^{m_\ell}(\theta,\phi)e^{-\imath\omega t}$

$\psi(r,\theta,\phi) = R_{n\ell}(r)Y_\ell^{m_\ell}(\theta,\phi)$

$|\vec{L}| = \sqrt{\ell(\ell+1)}\hbar \qquad L_z = m_\ell\hbar$

$\ell \in \mathbb{N}^* \qquad 0 \le \ell \le n-1 \qquad m_\ell \in \mathbb{Z} \qquad -\ell \le m_\ell \le \ell$

9 Atomic Structure

$\vec{\mu} = \frac{q}{2m}\vec{L} \qquad \mu_z = -\mu_\mathrm{B}m_\ell$

$U_\mathrm{M} = -\vec{\mu}\cdot\vec{B} \qquad U_\mathrm{M} = \hbar\omega_\mathrm{L}m_\ell \qquad \omega_\mathrm{L} = \frac{e|\vec{B}|}{2m_\mathrm{e}}$

$S_z = s\hbar \qquad |\vec{S}| = \frac{\sqrt{3}}{2}\hbar \qquad \vec{\mu}_S = -\frac{e}{m_\mathrm{e}}\vec{S}$

References

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